WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

MTMADSE04T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5=10$

(a) Show that the equation $x^n+nx^{n-1}+n(n-1)x^{n-2}+\ldots+n!=0$ can not have equal roots.

(b) If $\alpha,\beta,\gamma,\delta$ are the roots of $x^4+px^3+qx^2+rx+s=0$ then find the value of $\sum\alpha^2\beta\gamma$.

(c) If α is a special root of $x^n-1=0$, then prove that $\frac{1}{\alpha}$ is also a special root of it.

(d) If α be a imaginary root of the equation $x^n-1=0$, and n be a prime number, then show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)\ldots.(1-\alpha^{n-1})=n$.

(e) Find the value of k for which $(x+1)^4+x^4+\frac{k}{2}=0$ is a reciprocal equation.

(f) If α be a special root of the equation $x^8-1=0$, prove that

$$1+3\alpha+5\alpha^2+\ldots\ldots\ldots\ldots+15\alpha^7=\frac{16}{\alpha-1}$$

(g) Show that for real values of μ, the equation $(x+1)(x-3)(x-5)(x-7)+\mu(x-2)(x-4)(x-6)(x-8)=0$ has all its roots real and simple.

(h) Find the equation of fourth degree with real coefficient one root of which is $\sqrt{2+i\sqrt{3}}$.

2. (a) Find the special roots of the equation $x^{24}-1=0$ and hence deduce that $\cos\frac{\pi}{12}=\frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\cos\frac{5\pi}{12}=\frac{\sqrt{3}-1}{2\sqrt{2}}$. 4

(b) A polynomial $f(x)$ leaves the remainders 10 and $2x-3$ when it is divided by $(x-2)$ and $(x+1)^2$ respectively. Find the remainder when it is divided by $(x-2)(x+1)^2$. 4

3. (a) Show that the equation $x^4 - 14x^2 + 24x + k = 0$ has two distinct real roots if $-8 < k < 117$. 4

   (b) Applying Strum's theorem show that the equation $x^4 - 12x^2 + 4 = 0$ has all roots are real and distinct. 4

4. (a) Show that the roots of the equation $\frac{1}{x+a_1} + \frac{1}{x+a_2} + \dots + \frac{1}{x+a_n} = \frac{1}{x+b}$ are all real, where $a_1, a_2, \dots a_n$ and b are all positive real numbers and $b > a_i$ for all i. 4

   (b) Solve the cubic $x^3 - 27x - 54 = 0$ by Cardan's method. 4

5. (a) Applying Newton's theorem find the sum of 5^{th} powers of the roots of the equation $x^3 + qx + r = 0$. 4

   (b) If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has three equal roots, then show that $ab^4 - 9a^5 + c^5 = 0$. 4

6. (a) If the two polynomials $f(x)$ and $g(x)$, both of degree n take equal values for more than n distinct values of x, then $f(x)$ and $g(x)$ are identical polynomials. 3

   (b) Solve the equation $x^4 + 5x^3 + x^2 - 13x + 6 = 0$ by Ferrari's method. 5

7. (a) Solve $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$. 4

   (b) If n be a prime number prove that the special roots of the equation $x^{2n} - 1 = 0$ are the non real roots of the equation $x^n + 1 = 0$. 4

8. (a) Show that the condition that the sum of two roots of the equation $x^4 + mx^2 + nx + p = 0$ is equal to the product of the other two roots is $(2p - n)^2 = (p - n)(p + m - n)^2$. 4

   (b) Find the values of k for which the equation $x^3 - 9x^2 + 24x + k = 0$ may have multiple roots and solve the equation in each case. 4

9. (a) Find an upper limit of the real roots of the equation $x^4 + 4x^3 - 11x^2 - 9x - 50 = 0$. 4

   (b) If α, β, γ be the roots of $x^3 + qx + r = 0$, prove that $6S_5 = 5S_2S_3$, where $S_r = \Sigma\alpha^r$. 4

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